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Bi-maximal Neutrino Mixing With SO(3) Flavour Symmetry

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We demonstrate that an $SU(2)_L \times U(1)_Y$ model with extended Higgs sector gives rise to bi-maximal neutrino mixing through the incorporation of SO(3) flavour symmetry and discrete symmetry. The neutrino and the charged lepton masses are generated due to higher dimensional terms. The hierarchical structures of the masses of neutrinos and charged leptons are obtained due to inclusion of SO(3) flavour symmetry and discrete symmetry. The model can accommodate both the solutions of solar neutrino problem, vacuum oscillation or large angle MSW, through reasonable choice of model parameters along with the atmospheric neutrino experimental result.

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Evidence in favour of neutrino oscillation (as well as neutrino mass) has been provided by the Super-Kamiokande (SK) atmospheric neutrino experiment [1] through the measurement of magnitude and angular distribution of the ν_μ flux produced in the atmosphere due to cosmic ray interactions. Observed depletion of ν_μ flux in earth has been interpreted as the oscillation of ν_μ to some other species of neutrino. In a two flavour neutrino oscillation scenario, oscillation between ν_μ - ν_τ, the experimental result leds to maximal mixing between two species $\mathrm{Sin}^2 2\theta \sim 0.82$ with a mass-squared difference $\Delta m^2_{atm} \sim (5\times10^{-4} - 6\times10^{-3})$ eV^2. The solar neutrino experimental results [2] are also in concordance with the interpretation of atmospheric neutrino experimental result and the data provide the following values as $\Delta m^2_{e\mu} \sim (0.8-2)\times10^{-5}\mathrm{eV}^2$, $\mathrm{Sin}^2 2\theta \sim 1$ (Large angle MSW solution) or $\Delta m^2_{e\mu} \sim (0.5-6)\times10^{-10}\mathrm{eV}^2$, $\mathrm{Sin}^2 2\theta \sim 1$ (vacuum oscillation solution). Furthermore, the CHOOZ experimental result [3] gives the value of $\Delta m^2_{eX} < 10^{-3}$ eV^2 or $\mathrm{Sin}^2 2\theta_{\mathrm{eX}} < 0.2$. In order to reconcile with the solar and atmospheric neutrino experimental results, a distinct pattern of neutrino mixing emerges, namely, bi-maximal neutrino mixing [4], in which θ_{12}=$\theta_{23} = 45^o$, and if, the CHOOZ experimental result is interpreted in terms of $\nu_e - \nu_\tau$ oscillation, then $\theta_{31} < 13^o$.

In the prsent work, we demonstrate that an $\mathrm{SU(2)_L \times U(1)_Y}$ model wih extended Higgs sector coupled with an SO(3) flavour symmetry [5,6] and discrete $\mathrm{Z_3 \times Z_3' \times Z_4}$ symmetry, gives rise to bi-maximal neutrino mixing along with the value of θ_{31} very small. Instead of three almost degenerate neutrinos [5,7], we obtain a hierarchical pattern of neutrino masses. The charged lepton masses are also hierarchical in nature and this is due to the inclusion of SO(3) flavour symmetry, which, when spontaneously broken, gives rise to the desired hierarchy in mass [6]. The discrete $\mathrm{Z_3 \times Z_3' \times Z_4}$ symmetry also plays the crucial role to obtain the required mixing angle by prohibiting unwanted mass terms in the neutrino and charged lepton mass matrices. However, We consider soft discrete symmetry breaking terms in the scalar potential, which are also responsible to obtain non-zero values of the VEV's of the Higgs fields upon minimization of the scalar potential. The Majorana neutrino masses are obtained due to explicit breaking of lepton number through higher dimensional terms. The leptonic fields ($\mathrm{l_{iL}, E_{iR}}$, i = 1, 2, 3 is the generation index) and the Higgs fields ($\chi, \xi, \phi_1, \phi_2, \phi_3, \xi_e, \xi_\mu, \mathrm{h}$) have the following representation contents :

$$
\begin{aligned}
\mathrm{l_{iL}}\ (1,2,-1),\ \ \mathrm{E_R}\ (3,1,-2),\ \ \chi\ (3,1,0),\ \ \xi\ (3,1,0),\\
\phi_1\ (3,1,0),\ \ \phi_2\ (3,1,0),\ \ \phi_3\ (3,1,0),\ \ h\ (1,2,1),\\
\xi_e\ (1,1,0),\ \xi_\mu\ (1,1,0)
\end{aligned}
\tag{1}
$$

where the digits in the parentheses represent SO(3), $\mathrm{SU(2)_L}$ and $\mathrm{U(1)_Y}$ quantum numbers. The subscript of the ϕ Higgs fields denote the direction of development of non-zero VEV, such as $<\phi_1> = (v_1, 0, 0)$ etc. and the subscript below ξ fields denote the respective coupling with the charged leptons. Regarding Higgs content and the symmetry breaking pattern, the present model is analogous to a supersymmetric model discussed in Ref.[6], where the Higgs scalars are replaced by 'flavon' chiral superfields. Regarding the representation content of leptonic fields, $\mathrm{l_{iL}(E_{iR})}$ are triplet(singlet) under SO(3) gauge group in Ref.[6], just opposite to our case. We consider the following discrete $Z_3 \times Z_3' \times Z_4$ symmetry transformation of the lepton and Higgs fields, in order to achieve bi-maximal neutrino mixing:

$\underline{Z_3 \times Z_3' \times Z_4 \text{ Symmetry}}$

$$\begin{aligned} \mathrm{l_{1L}} \to \mathrm{i}\alpha\omega \mathrm{l_{1L}},\ \ \mathrm{l_{2L}} \to -\mathrm{il_{2L}},\ \ \mathrm{l_{3L}} \to -\mathrm{il_{3L}},\ \ \mathrm{E_R} \to \mathrm{i}\alpha \mathrm{E_R} \\ \chi \to \alpha\omega\chi,\ \ \xi \to i\xi,\ \ \mathrm{h} \to \mathrm{h},\ \ \phi_1 \to \alpha\omega^2\phi_1,\ \ \phi_2 \to \alpha^2\omega\phi_2, \\ \phi_3 \to \alpha^2\phi_3,\ \ \xi_e \to \alpha^2\omega^2\xi_e,\ \ \xi_\mu \to -\alpha^2\omega\xi_\mu \end{aligned} \tag{2}$$

where ω and α are the generators of $\mathrm{Z_3}$ and $\mathrm{Z_3'}$ group, respectively. The most general lepton-Higgs Yukawa interaction in the present model generating Majorana neutrino masses is given by

$$\begin{aligned} L_Y^\nu = \frac{(\mathrm{l_{1L}l_{2L}})(\chi\chi)\mathrm{hh}}{\mathrm{M_f^3}} + \frac{(\mathrm{l_{1L}l_{3L}})(\chi\chi)\mathrm{hh}}{\mathrm{M_f^3}} + \frac{(\mathrm{l_{2L}l_{2L}})(\xi\xi)\mathrm{hh}}{\mathrm{M_f^3}} \\ + \frac{(\mathrm{l_{2L}l_{3L}})(\xi\xi)\mathrm{hh}}{\mathrm{M_f^3}} + \frac{(\mathrm{l_{3L}l_{3L}})(\xi\xi)\mathrm{hh}}{\mathrm{M_f^3}} \end{aligned} \tag{3}$$

and the Yukawa interaction which is responsible for generation of charged lepton masses is given by

$$\begin{aligned} L_Y^E = \frac{(\mathrm{e_R}.\phi_1)\mathrm{l_{2L}h}\xi_\mathrm{e}^2}{\mathrm{M_f^3}} + \frac{(\mathrm{e_R}.\phi_1)\mathrm{l_{3L}h}\xi_\mathrm{e}^2}{\mathrm{M_f^3}} + \frac{(\mathrm{e_R}.\phi_2)\mathrm{l_{1L}h}\xi_\mu}{\mathrm{M_f^2}} \\ + \frac{(\mathrm{e_R}.\phi_3)\mathrm{l_{2L}h}}{\mathrm{M_f}} + \frac{(\mathrm{e_R}.\phi_3)\mathrm{l_{3L}h}}{\mathrm{M_f}}. \end{aligned} \tag{4}$$

In the above Lagrangian, we consider all the couplings are equal to unity and all the VEV's are real. The present model contains a large mass scale M_f, and for our analysis we set $\mathrm{M_f} \sim \mathrm{M_{GUT}}$. We also consider that the flavour symmetry group SO(3) is broken below the GUT scale , but much above the electroweak scale corresponds to $< \mathrm{h} >$. On the otherway, the scale of SO(3) symmetry breaking VEV's, $< \chi >$ and $< \xi >$ are constrained by the solar and atmospheric neutrino experimental results, and the VEV's of $< \phi_1 >$,

$< \phi_2 >$, $< \phi_3 >$, $< \xi_e >$ and $< \xi_\mu >$ determine the masses of the charged leptons, which in turn constrain the θ_{31} mixing angle. The Higgs fields ξ_e, ξ_μ are singlet under the gauge symmetry and their VEV's in principle can take values above the SO(3) symmetry breaking scale.

In order to avoid any zero values of the VEV's of the Higgs fields upon minimization of the scalar potential, we have to consider discrete symmetry breaking terms. Without going into the details of the scalar potential, this feature can be realized in the following way. In general, the scalar potential can be written as (keeping upto dim=4 terms)

$$\mathrm{V} = \mathrm{A}\mathrm{y}^4 + \mathrm{B}\mathrm{y}^3 + \mathrm{C}\mathrm{y}^2 + \mathrm{D}\mathrm{y} + \mathrm{E} \tag{5}$$

where 'y' is the VEV of any Higgs field and A, B, C, D, E are generic couplings of the terms contained in the scalar potential. Minimizing the scalar potential w.r.t. 'y', we obtain

$$\mathrm{V}' = \mathrm{A}'\mathrm{y}^3 + \mathrm{B}'\mathrm{y}^2 + \mathrm{C}'\mathrm{y} + \mathrm{D} \tag{6}$$

Eqn.(6) reflects the fact that as long as $\mathrm{D} \neq 0$, and A' or B' or C' is not equal to zero, we will get non-zero solutions for 'y'. Thus, in order to obtain $y \neq 0$ solution, it is necessary to retain the terms with generic coefficients D and A' or B' or C'. In the present model, both the discrete symmetry breaking terms soft and hard, correspond to the term with coefficient D. Dicarding hard symmetry breaking terms, we retain soft discrete symmetry breaking terms, and, hence, none of the VEV is zero upon minimization of the scalar potential.

Let us look at the charged lepton sector. Substituting the VEV's of the Higgs fields appeared in Eqn.(4), we obtain the charged lepton mass matrix given by

$$M_E = \begin{pmatrix} 0 & d & d \\ e & 0 & 0 \\ 0 & f & f \end{pmatrix} \tag{7}$$

where $d = \frac{<\phi_1><\mathrm{h}><\xi_\mathrm{e}>^2}{M_f^3}$, $e = \frac{<\phi_2><\mathrm{h}><\xi_\mu>}{M_f^2}$ and $f = \frac{<\phi_3><\mathrm{h}>}{M_f}$. The hierarchy between the d, e and f parameters, $d < e < f$ is manifested due to the large mass scale M_f. Diagonalizing M_E^2 , we obtain the following eigenvalues and mixing angles as

$$\mathrm{m}_{\mathrm{E}_1}^2 = \sqrt{2}\mathrm{d}$$

$$\mathrm{m}_{\mathrm{E}_2}^2 = \mathrm{e}$$

$$\mathrm{m}_{\mathrm{E}_3}^2 = \sqrt{2}\mathrm{f} \tag{8}$$

and $\theta_{12}^E = \theta_{23}^E = 0$, $\tan 2\theta_{31}^{\mathrm{E}} = \frac{2\mathrm{df}}{\mathrm{f}^2-\mathrm{d}^2}$. The zero values of θ_{12}^E, θ_{23}^E is assured due to discrete symmetry invariance. The hierarchy in the charged lepton masses as well as small values of $\tan 2\theta_{31}^{\mathrm{E}} \sim 0$ arises due to the hierarchy already manifested in the mass matrix given in Eqn.(5). The number of parameters in M_E are large enough to fit the three eigenvalues with the masses of the three charged leptons, as well as to keep $\tan 2\theta_{31}^{\mathrm{E}}$ well within the experimental value given by CHOOZ experiment as mentioned earlier.

Let us now focus our attention to the neutrino sector of the model. Substituting the VEV's of χ, ξ and h Higgs fields in Eqn.(3), we get the Majorana neutrino mass matrix as follows:

$$M_\nu = \begin{pmatrix} 0 & a & a \\ a & b & b \\ a & b & b \end{pmatrix} \tag{9}$$

where $a = \frac{<\chi>^2<\text{h}>^2}{M_f^3}$, $b = \frac{<\xi>^2<\text{h}>^2}{M_f^3}$. It is to be noted that the absence of $\nu_e\nu_e$ mass term in the above mass matrix (at the tree level) evades the bound on Majorana neutrino mass due to $\beta\beta_{o\nu}$ decay. Diagonalizing the neutrino mass matrix M_ν by an orthogonal transformation , we obtain the following values of the mixing angles, $\theta_{12}^\nu = \theta_{23}^\nu = \frac{\pi}{4}$, and $\theta_{31}^\nu = 0$. Thus the present model gives rise to bi-maximal neutrino mixing pattern as $\theta_{12} = \theta_{23} = \frac{\pi}{4}$, and $\theta_{31} \sim 0$ where θ_{12}, θ_{23} and θ_{31} are the mixing angles appeared in the charged current neutrino - lepton interactions. The eigenvalues of the above mass matrix comes out as

$$\begin{aligned} m_{\nu_1}^2 &= 0 \\ m_{\nu_2}^2 &= b + x \\ m_{\nu_3}^2 &= b - x \end{aligned} \tag{10}$$

where $x = \sqrt{b^2 + 2a^2}$. The value of x depends on the hierarchical relation between a and b parameters which is manifested from the values of $\Delta m_{23}^2 = 4bx$ and $\Delta m_{21}^2 = (b + x)^2$. Now, if, $b > a$, then the value of x comes out as $x = b(1 + \frac{a^2}{b^2}) \sim b$ and, hence, $\mathrm{m}_{\nu_1} = \mathrm{m}_{\nu_2} = 0$ and $m_{\nu_3} = 2b$. Then both the mass squared differences are parametrized in terms of a single parameter b, and, hence, in this case it is not possible to accommodate both the results of solar and atmospheric neutrino experiments. The same scenario appears for $a = b$ case, and, hence, for a phenomenologically viable model, we have to consider the third option $a > b$. In this situation, we obtain, $\Delta m_{23}^2 = 4ab$, $\Delta m_{21}^2 = (a + b)^2 \sim 2a^2$. For a typical value of $\Delta m_{23}^2 \sim 4 \times 10^{-3}$ eV^2 which can explain the atmospheric neutrino deficits, we obtain $2a^2 \sim 6 \times 10^{-3}$ eV^2 which in turn gives rise to the value of $< \chi > \sim 10^{11}$ GeV for $M_f \sim 10^{12}$ GeV and $<$ h $> \sim 100$ GeV. Using the same values of M_f and $< h >$ parameters, it is possible to accommodate the solar neutrino experimental result for both types of solutions, vacuum oscilation and large angle MSW for two different values of parameter b. For a typical value of $\Delta m^2_{23}{\sim}4\times10^{-10}$ eV^2 which can explain the solar neutrino deficits due to vacuum oscillation, the value of b^2 comes out as $b^2\sim 0.083\ \times10^{-17}$eV^2 which leds to the value of $<\xi>\sim10^7$ GeV and for large angle MSW solution, a typical value of $\Delta m^2_{23}{\sim}10^{-5}$eV^2 gives rise to $b^2\sim10^{-9}$eV^2 and $<\xi>\sim 2\times10^9$ GeV.

In summary, we demonstrate that an $SU(2)_L\times U(1)_Y$ model with an extended Higgs sector, SO(3) flavour symmetry and discrete $Z_3\times Z_3'\times Z_4$ symmetry, gives rise to bi-maximal neutrino mixing $\theta_{12}=\theta_{23}=\frac{\pi}{4}$ consistent with the present solar and atmospheric neutrino experimental results. Neutrino masses are generated due to explicit lepton number violating higher dimensional terms (dim=7) and the charged lepton masses are generated due to dim=5,6,7 terms. Furthermore, the hierarchy obtained in the charged lepton masses is also responsible to keep the mixing angle θ_{31} to be very small so as to satisfy the CHOOZ experimental result. The discrete symmetry plays the crucial role to obtain the bi-maximal neutrino mixing as well as prohibits $\nu_e\nu_e$ mass term in the neutrino mass matrix at the tree level so as to evade the bound on the Majorana neutrino mass from $\beta\beta_{0\nu}$ decayi in the present model. The vacuum neutrino oscillation solution or the large angle MSW solution can be achieved in the present model for a reasonable choice of model parameters along with the atmospheric neutrino experimental results.

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